

The Regular Universe

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Abstract

A regular (i.e., singularity-free) cycling cosmological model is advanced. In the model, there are only two constants: the gravitational constant (or the Planck time) and the cosmic period. The radius of the universe is a simple periodic function of cosmic time. The regularity of the construction is achieved via the condition that the measure associated with metric be the Haar measure on the 3-space. The possibility of the construction is due to dark matter—as long as it is treated not as a particle matter, but as a compensational tensor field. The metrodynamical equation, i.e., an equation for metric, is derived and expressions for the pressure, energy and momentum compensations are obtained.

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Introduction

If a physical theory contains singularities, this means that the theory may be incomplete. As long as the source of a metric field in General Relativity is treated as an ordinary matter, this results in singularities of metric and of the matter distribution—both local (black holes) and global, or cosmic (the Big Bang). It appears natural to try to get rid of the singularities, i.e., to construct a complete, or regular theory of the universe.

The regularity of the universe means above all the regularity of the spacetime manifold. The latter is a pair (M^4, g) [1] where M^4 is a connected four-dimensional Hausdorff C^∞ manifold and g is a Lorentz metric on M^4 . The Lorentz metric is regular, i.e., nonsingular, which means that its determinant is not equal to zero. However, the determinant depends on a coordinate system; therefore, using the determinant directly, it is impossible to formulate the nonsingularity condition geometrically, i.e., in a coordinate-free way, and to incorporate the condition into metric dynamics.

A coordinate-free formulation of the nonsingularity condition may be achieved by means of the measure associated with metric. Metric of the universe induces the Riemannian metric on the 3-sphere, which may be represented in the form of $R^2(t)h_{ij}(t, \underline{x})dx^i dx^j$ with a preassigned radius of the universe $R(t)$.

The nonsingularity condition: The measure associated with the metric h_{ij} is the Haar measure [2] on the 3-sphere. It is this condition that provides for the absence of singularities.

A possibility of such a construction arises in connection with dark matter—if it is treated not as a particle matter [3], but rather as a compensational tensor field, or the compenson [4]. A pivotal idea of the construction is the following. Since the compenson is not a particle field, there exists no equation for the compenson proper. An equation in addition to the Einstein equations should be introduced for metric. The additional equation is the result of the nonsingularity condition and it provides for the absence of singularities.

The resulting construction describes the regular, i.e., singularity-free cycling universe.

As to the absence of black holes, it should be pointed out that a contracting-expanding (or pulsating) massive star may simulate the behavior of a black hole: an external observer will see the star reach the horizon only after an infinitely long time [5]. A pure compenson object may appear to be a black hole as well.

A compensating term in the dynamical (i.e., spatial) components of the extended Einstein equation is represented in the simplest form—as a pressure compenson. This makes it possible to eliminate the term from the equation, which results in the metrodynamical equation—a dynamical equation for metric. The Haar measure condition may be regarded as an integral of motion of the metrodynamical equation.

The pressure, energy and momentum compensons are represented in an explicit form.

1 Compenson

1.1 The extended Einstein equation. Compenson

The extended Einstein equation,

$$G_\mu^\nu = 8\pi\kappa(T_\mu^\nu + \Theta_\mu^\nu), \quad \mu, \nu = 0, 1, 2, 3 \quad (1.1.1)$$

includes apart from the matter energy-momentum tensor

$$T_\mu^\nu = (\Psi, \hat{T}_\mu^\nu \Psi) \quad (1.1.2)$$

the compenson, i.e., a compensational tensor field Θ_μ^ν . In (1.1.1), G_μ^ν is the Einstein tensor and $\varkappa = t_{\text{Planck}}^2$ is the gravitational constant ($c = \hbar = 1$). The state vector Ψ is guided by the Schrödinger equation.

1.2 Synchronous gauge

Spacetime manifold is

$$M^4 = T \times S^3 \quad (1.2.1)$$

where T is cosmic time and the 3-sphere S^3 is cosmic space. Correspondingly, metric is of the form

$$ds^2 = dt^2 + g_{ij} dx^i dx^j, \quad i, j = 1, 2, 3 \quad (1.2.2)$$

(synchronous [5], or temporal [6] gauge). Here (x^1, x^2, x^3) are dimensionless coordinates on S^3 and the dimensions are these:

$$[g_{ij}] = [t^2] = T^2 \quad (1.2.3)$$

1.3 Stress, energy and momentum compensons

Any tensor splits into the spatial, energy and momentum components:

$$A_\mu^\nu = (A_i^j, A_0^0, A_i^0) \quad (1.3.1)$$

and equation (1.1.1) splits accordingly:

$$G_i^j = 8\pi\varkappa(T_i^j + \Theta_i^j) \quad (1.3.2)$$

$$G_0^0 = 8\pi\varkappa(T_0^0 + \Theta_0^0) \quad (1.3.3)$$

$$G_i^0 = 8\pi\varkappa(T_i^0 + \Theta_i^0) \quad (1.3.4)$$

Here Θ_i^j , Θ_0^0 and Θ_i^0 are the stress, energy and momentum compensons, respectively.

Now, (1.3.2) is a system of dynamical equations, and (1.3.3), (1.3.4) determine the energy-momentum compenson:

$$\Theta_\mu^0 = \frac{1}{8\pi\varkappa} G_\mu^0 - T_\mu^0 \quad (1.3.5)$$

1.4 The cosmological constant, dark energy, and dark matter

Let us establish the relation of our treatment with the conventional one. Rewrite (1.1.1) in the form

$$G_\mu^\nu - \Lambda g_\mu^\nu = 8\pi\varkappa(T_\mu^\nu + T_{\text{dark matter}}^\nu) \quad (1.4.1)$$

where Λ is the cosmological constant and

$$T_{\text{dark matter}}^\nu := \Theta_\mu^\nu - \frac{\Lambda}{8\pi\varkappa} g_\mu^\nu \quad (1.4.2)$$

is the dark matter energy-momentum tensor.

Next,

$$G_0^0 = 8\pi\kappa(\varrho_{\text{matter}} + \varrho_{\text{dark matter}} + \varrho_{\text{dark energy}}) \quad (1.4.3)$$

where

$$\varrho_{\text{matter}} = T_0^0 \quad (1.4.4)$$

$$\varrho_{\text{dark matter}} = T_{\text{dark matter}0}^0 = \Theta_0^0 - \frac{\Lambda}{8\pi\kappa} \quad (1.4.5)$$

$$\varrho_{\text{dark energy}} = \frac{\Lambda}{8\pi\kappa} \quad (1.4.6)$$

2 Radius of the universe, the Haar measure condition, and dynamical equations

2.1 Radius introduced

Put

$$g_{ij}(t, \underline{x}) = -R^2(t)h_{ij}(t, \underline{x}), \quad \underline{x} := (x^1, x^2, x^3) \in S^3 \quad (2.1.1)$$

with the dimensions

$$[R] = T, \quad [h_{ij}] = T^0 \quad (2.1.2)$$

so that

$$ds^2 = dt^2 - R^2(t)h_{ij}(t, \underline{x})dx^i dx^j \quad (2.1.3)$$

Here $R(t)$ is the radius of the universe, which is subject to specification, and h_{ij} is the Riemannian metric on S^3 , which is determined by dynamical equations.

2.2 Stress compenson

The compenson is not a particle field, therefore there exist no equations for the compenson proper. The energy-momentum compenson is determined by (1.3.3), (1.3.4). But the stress compenson Θ_i^j is contained in dynamical equations (1.3.2), so that it appears that a natural approach is to introduce additional equations for metric. To minimize the number of the equations, Θ_i^j should be reduced to a single scalar function ϑ .

Put

$$\Theta_i^j = \Theta_i^j[\vartheta] \quad (2.2.1)$$

2.3 The Haar measure condition

In (1.3.2), there are 6 equations for 7 quantities h_{ij} , ϑ , so that it is necessary to introduce one more equation for metric h_{ij} .

Introduce a condition on the measure associated with the metric h_{ij} ,

$$d\mu_t(\underline{x}) = \sqrt{|h|}d\underline{x} = \sqrt{|h|}dx^1 dx^2 dx^3, \quad |h| = \det(h_{ij}) \quad (2.3.1)$$

The condition is this: (2.3.1) is the Haar measure [3] on S^3 . The latter is associated with the metric [8]

$$d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2), \quad 0 \leq \chi \leq \pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \varphi \leq 2\pi \quad (2.3.2)$$

and is

$$d\mu(\omega) = \sin^2 \chi \sin \theta d\chi d\theta d\varphi, \quad \omega = (\chi, \theta, \varphi) \quad (2.3.3)$$

Thus, the condition is that (2.3.1) coincide with (2.3.3):

$$d\mu_t(\underline{x}) = d\mu(\omega) \quad (2.3.4)$$

hence

$$\frac{\partial}{\partial t} |h| = 0 \quad (2.3.5)$$

Next, it holds that [8]

$$\frac{\partial}{\partial t} \ln |h| = h^{ij} \dot{h}_{ij}, \quad \cdot = \frac{\partial}{\partial t} \quad (2.3.6)$$

thus,

$$h^{ij} \dot{h}_{ij} = 0 \quad (2.3.7)$$

Equations (1.3.2) for h_{ij} are of the second order in t , so we pass on to the equation

$$\frac{\partial}{\partial t} (h^{ij} \dot{h}_{ij}) = 0 \quad (2.3.8)$$

i.e.,

$$h^{ij} \ddot{h}_{ij} + \dot{h}^{ij} \dot{h}_{ij} = 0 \quad (2.3.9)$$

Now (2.3.4) and (2.3.7) are constraints.

Note that equations (2.3.9), (2.3.4), (2.3.7) are 3-covariant (coordinate-free).

2.4 Dynamical equations

Dynamical equations for metric h_{ij} and the stress compenson function ϑ are

$$G_i^j - 8\pi \varkappa \Theta_i^j[\vartheta] = 8\pi \varkappa T_i^j \quad (2.4.1)$$

$$h^{ij} \ddot{h}_{ij} + \dot{h}^{ij} \dot{h}_{ij} = 0 \quad (2.4.2)$$

Initial conditions are for h_i^j , \dot{h}_i^j with constraints (2.3.4), (2.3.7). The solution may be obtained via a step by step procedure.

(2.3.4) is the regularity condition.

3 Radius specified

3.1 Observational constraints on the radius

There are two quantities associated with the radius $R(t)$, the present-day values of which are known from observations: the Hubble parameter

$$H = \frac{\dot{R}}{R} \quad (3.1.1)$$

and the deceleration

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{\ddot{R}}{RH^2} \quad (3.1.2)$$

The present-day values are [9]

$$H = (2.28 \pm 0.05) \times 10^{-18} \text{ sec}^{-1}, \quad q = -0.595 \pm 0.025 \quad (3.1.3)$$

A specified $R(t)$ should be compatible with those values.

3.2 Radius specification

Put

$$R(t) = \sqrt{\varkappa} + (R_{\max} - \sqrt{\varkappa})f(t) \quad (3.2.1)$$

where

$$R_{\max} \gg \sqrt{\varkappa} = t_{\text{Planck}} \quad (3.2.2)$$

and

$$f(t+T) = f(t) \text{ for some } T \quad (3.2.3)$$

$$f(-t) = f(t), \quad f(0) = 0, \quad \dot{f}(0) = 0, \quad f(T/2) = 1 \quad (3.2.4)$$

We have

$$f(T/2 + \tau) = f(-T/2 - \tau) = f(-T/2 - \tau + T) = f(T/2 - \tau) \quad (3.2.5)$$

i.e.,

$$f(T/2 - \tau) = f(T/2 + \tau), \quad \dot{f}(T/2) = 0 \quad (3.2.6)$$

Next, put

$$f(0) = f_{\min}, \quad f(T/2) = f_{\max} \quad (3.2.7)$$

so that

$$R(0) = R_{\min} = t_{\text{Planck}} \ll R_{\max} = R(T/2) \quad (3.2.8)$$

Consider the case

$$0 < t < T/2 \text{ and } R_{\max}f(t) \gg \sqrt{\varkappa}, \quad R(t) \cong R_{\max}f(t) \quad (3.2.9)$$

Try the simplest f :

$$f(t) = \sin^2 \nu t, \quad \nu = \pi/T \quad (3.2.10)$$

We have

$$R(t) = R_{\max} \sin^2 \nu t, \quad \dot{R}(t) = R_{\max} 2\nu \sin \nu t \cos \nu t, \quad \ddot{R}(t) = R_{\max} 2\nu^2 (\cos^2 \nu t - \sin^2 \nu t) \quad (3.2.11)$$

and

$$q = -\frac{1}{2} + \frac{1}{2} \tan^2 \nu t > -0.5 \quad (3.2.12)$$

Thus, in view of (3.1.3), (3.2.10) is not suitable.

The next candidate is

$$f(t) = \sin^4 \nu t, \quad \nu = \pi/T \quad (3.2.13)$$

Now

$$R(t) = R_{\max} \sin^4 \nu t, \quad \dot{R}(t) = R_{\max} 4\nu \sin^3 \nu t \cos \nu t, \quad \ddot{R}(t) = R_{\max} 4\nu^2 \sin^2 \nu t (3 \cos^2 \nu t - \sin^2 \nu t) \quad (3.2.14)$$

and

$$q = -\frac{3}{4} + \frac{1}{4} \tan^2 \nu t \quad (3.2.15)$$

so that

$$\frac{1}{4} \tan^2 \nu t = 0.75 - 0.595 \pm 0.025 = 0.155 \pm 0.025 \quad (3.2.16)$$

Thus, (3.2.13) is plausible.

3.3 Parameters determined

We have two equations:

$$\tan^2 \nu t = 3 + 4q \quad (3.3.1)$$

$$\frac{4\nu}{\tan \nu t} = H \quad (3.3.2)$$

Hence

$$\nu = \frac{1}{4} H \sqrt{3 + 4q} \quad (3.3.3)$$

$$t = \frac{1}{\nu} \tan^{-1} \sqrt{3 + 4q}, \quad t = t_0 := t_{\text{present}} \quad (3.3.4)$$

T is the cosmic period and ν is the cosmic frequency.

We find

$$\nu = 0.449 \times 10^{-18} \text{ sec}^{-1}, \quad T = 7.00 \times 10^{18} \text{ sec} = 222 \times 10^9 \text{ yr} \quad (3.3.5)$$

Since t_{Planck} is a dimensional quantity, there should be another parameter with the same dimensionality—in order that small and large make sense. It is T that is the large counterpart:

$$t_{\text{Planck}} = 5.39 \times 10^{-44} \text{ sec} \lll 7.00 \times 10^{18} \text{ sec} = T \quad (3.3.6)$$

There are only two constants in the model: the gravitational constant

$$\varkappa = t_{\text{Planck}}^2 \quad (3.3.7)$$

and the cosmic frequency, or

$$\nu^2 = (\pi/T)^2 \quad (3.3.8)$$

Correspondingly, there is a natural dimensionless constant

$$\varkappa\nu^2 = 5.86 \times 10^{-124} \lll 1 \quad (3.3.9)$$

Put

$$R_{\max} = \frac{R_{\min}}{\varkappa\nu^2} = \frac{\sqrt{\varkappa}}{\varkappa\nu^2} = \frac{1}{\nu(\sqrt{\varkappa}\nu)} \quad (3.3.10)$$

We obtain

$$R_{\max} = 0.920 \times 10^{80} \text{ sec} \quad (3.3.11)$$

and

$$t_0 := t_{\text{present}} = 1.49 \times 10^{18} \text{ sec} = 47.2 \times 10^9 \text{ yr}, \quad R_0 := R_{\text{present}} = 0.136 \times 10^{80} \text{ sec} \quad (3.3.12)$$

(Without regard to the compenson, $t_0 = 13,7 \times 10^9 \text{ yr}$ [9].)

Finally, introduce the inflection instant:

$$t_{\text{inflection}}, \quad \ddot{R}(t_{\text{inflection}}) = 0 \quad (3.3.13)$$

so that

$$\ddot{R}(t) \begin{cases} > 0 & \text{for } -t_{\text{inflection}} < t < t_{\text{inflection}} \\ < 0 & \text{for } t_{\text{inflection}} < t < T - t_{\text{inflection}} \end{cases} \quad (3.3.14)$$

and

$$q(t_{\text{inflection}}) = 0 \quad (3.3.15)$$

We find

$$t_{\text{inflection}} = 2.34 \times 10^{18} \text{ sec} = 74.1 \times 10^9 \text{ yr} \quad (3.3.16)$$

So,

$$\begin{aligned} R_{\min} &\leftrightarrow t = 0 \\ &< t_{\text{present}} = 1.49 \times 10^{18} \text{ sec} = 47.2 \times 10^9 \text{ yr} \\ &< t_{\text{inflection}} = 2.34 \times 10^{18} \text{ sec} = 74.1 \times 10^9 \text{ yr} \\ &< T/2 = 3.50 \times 10^{18} \text{ sec} = 111 \times 10^9 \text{ yr} \leftrightarrow R_{\max} \end{aligned} \quad (3.3.17)$$

3.4 The first ten minutes

Put

$$t = 10 \text{ minutes} = 600 \text{ sec} \quad (3.4.1)$$

Since

$$\nu t \lll 1 \quad (3.4.2)$$

we have

$$\sin^4 \nu t = (\nu t)^4 = 52.7 \times 10^{-64} \quad (3.4.3)$$

and

$$R(t) = R_{\max}(\nu t)^4 = \frac{1}{\nu(\sqrt{\kappa\nu})}(\nu t)^4 = 0.218 \frac{1}{\nu} \quad (3.4.4)$$

Thus,

$$\frac{R(10 \text{ minutes})}{R(0)} = \frac{R(10 \text{ minutes})}{R_{\min}} = \frac{0.218}{\nu} / t_{\text{Planck}} = \frac{0.218}{\sqrt{\kappa\nu^2}} = 0.901 \times 10^{61} \quad (3.4.5)$$

4 The regular cycling universe

4.1 The aging problem

Although $R(t)$ is a periodic function, the construction of a cycling model is not yet complete. Such a model faces the aging problem [10,11]. We quote Weinberg [11]:

“Some cosmologists are philosophically attracted to the oscillating model, especially because . . . it nicely avoids the problem of Genesis. It does, however, face one severe theoretical difficulty. In each cycle the ratio of photons to nuclear particles . . . is slightly increased by a kind of friction (known as “bulk viscosity”) as the universe expands and contracts. As far as we know, the universe would then start each new cycle with a new, slightly larger ratio of photons to nuclear particles. Right now this ratio is large, but not infinite, so it is hard to see how the universe could have previously experienced an infinite number of cycles.”

4.2 Big Jump

A feasible resolution of the aging problem is via the following quantum jump:

$$\text{at } t = 0 \quad \Psi(t - 0) \rightarrow \Psi(t + 0) = \Psi_0, \quad \Psi_0 \text{ preassigned} \quad (4.2.1)$$

which may be called the Big Jump.

4.3 The regular cycling universe

Now we have a regular cycling model of the universe. Specifically,

$$R(t) = R_{\min} = \sqrt{\kappa} \quad \text{at } t = nT, \quad n = \dots, -2, -1, 0, 1, 2, \dots \quad (4.3.1)$$

$$R(t) = R_{\max} = \frac{1}{\nu\sqrt{\kappa\nu^2}} \quad \text{at } t = (n + 1/2)T, \quad n = \dots, -2, -1, 0, 1, 2, \dots \quad (4.3.2)$$

with

$$\text{at } t = nT \quad \Psi(nT - 0) \rightarrow \Psi(nT + 0) = \Psi_0, \quad \Psi_0 \text{ preassigned} \quad (4.3.3)$$

It must be emphasized that, in our treatment,

$$\text{regularity} \Rightarrow \text{cyclicity} \quad (4.3.4)$$

but, on the other hand,

$$\text{cyclicity} \not\Rightarrow \text{regularity} \quad (4.3.5)$$

So, the closed Friedmann model is oscillating but not regular (see [12] on cycling models).

4.4 Pseudo-black hole

Regularity implies the absence of black holes. But a contracting-expanding (or pulsating) massive star may simulate the behavior of a black hole. We quote Straumann [5]:

“An external observer far away from the star will see it reach the horizon only after an infinitely long time. As a result of the gravitational time dilation, the star “freezes” at the Schwarzschild horizon. However, in practice, the star will suddenly become invisible, since the redshift will start to increase exponentially..., and the luminosity decreases correspondingly.”

A pure compenson object may appear to be a black hole as well.

5 The isotropic universe

5.1 Metric and measure

Metric of the closed isotropic universe is of the form [8]

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\varphi^2)], \quad 0 \leq \chi \leq \pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \varphi \leq 2\pi \quad (5.1.1)$$

and the measure associated with it is (2.3.3):

$$d\mu(\omega) = \sin^2 \chi \sin \theta d\chi d\theta d\varphi, \quad \omega = (\chi, \theta, \varphi) \quad (5.1.2)$$

Thus, equations (2.3.4), (2.3.5), (2.3.7), (2.3.9) hold identically.

5.2 The extended Einstein equations

The extended Einstein equations are

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = 8\pi\kappa(T_1^1 + \Theta_1^1) \quad (5.2.1)$$

$$3\left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2}\right) = 8\pi\kappa(T_0^0 + \Theta_0^0) \quad (5.2.2)$$

where

$$R(t) = \sqrt{\kappa} + (R_{\max} - \sqrt{\kappa}) \sin^4 \nu t \quad (5.2.3)$$

5.3 Dynamical equation. Compenson pressure

There is only one dynamical equation (5.2.1), which determines the compenson pressure,

$$p_{\text{compenon}} := -\Theta_1^1 \quad (5.3.1)$$

$$p_{\text{compenon}} = -\frac{1}{8\pi\kappa} \left(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) - p_{\text{matter}} \quad (5.3.2)$$

where the matter pressure

$$p_{\text{matter}} = -T_1^1 \quad (5.3.3)$$

5.4 Compenson density

Equation (5.2.2), or

$$3 \left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) = 8\pi \varkappa (\varrho_{\text{matter}} + \varrho_{\text{compenson}}) \quad (5.4.1)$$

$$\varrho_{\text{matter}} = T_0^0, \quad \varrho_{\text{compenson}} = \Theta_0^0 \quad (5.4.2)$$

determines the compenson density:

$$\varrho_{\text{compenson}} = \frac{3}{8\pi \varkappa} \left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) - \varrho_{\text{matter}} \quad (5.4.3)$$

5.5 Matter density

We have

$$\varrho_{\text{dark energy}} + \varrho_{\text{dark matter}} + \varrho_{\text{matter}} \leftrightarrow 1 \quad (5.5.1)$$

and [9]

$$\text{at } t_0 = t_{\text{present}} \quad \varrho_{\text{dark energy}} = \frac{\Lambda}{8\pi \varkappa} \leftrightarrow 0.732, \quad \varrho_{\text{dark matter}} \leftrightarrow 0.223, \quad \varrho_{\text{matter}} \leftrightarrow 0.044 \quad (5.5.2)$$

with

$$\Lambda = 0.732 \times 3H_0^2 \quad (5.5.3)$$

Thus,

$$\text{at } t_0 = t_{\text{present}} \quad \varrho_{\text{matter}} = 0.044 \times \frac{\varrho_{\text{dark energy}}}{0.732} = \frac{3 \times 0.044}{8\pi} \frac{H_0^2}{\varkappa} \quad (5.5.4)$$

By (3.3.3),

$$H_0 = \frac{\nu}{0.197} \quad (5.5.5)$$

so that

$$\text{at } t_0 = t_{\text{present}} \quad \varrho_{\text{matter}} = \frac{0.044}{0.0388} \times \frac{3}{8\pi} \frac{\nu^2}{\varkappa} \quad (5.5.6)$$

Next,

$$\varrho_{\text{matter}}(0) \gtrsim \varrho_{\text{matter}}(t_0) \left(\frac{R_{\text{max}}}{\sqrt{\varkappa}} \right)^3 = \varrho_{\text{matter}}(t_0) \frac{1}{(\varkappa \nu^2)^3} \approx \frac{3}{8\pi} \frac{1}{\varkappa^2} \frac{1}{(\varkappa \nu^2)^2} = \frac{3}{8\pi} \frac{\varrho_{\text{Planck}}}{(\varkappa \nu^2)^2} \quad (5.5.7)$$

where

$$\varrho_{\text{Planck}} = \frac{1}{\varkappa^2} = \frac{1}{t_{\text{Planck}}^4} \quad (5.5.8)$$

Thus,

$$\varrho_{\text{matter}}(0) \sim \frac{\varrho_{\text{Planck}}}{(\varkappa \nu^2)^2} \quad (5.5.9)$$

5.6 Density parameters

Equation (1.4.3) reduces to

$$3 \left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) = 8\pi\kappa(\varrho_{\text{matter}} + \varrho_{\text{dark matter}} + \varrho_{\text{dark energy}}) \quad (5.6.1)$$

or

$$H^2 + \frac{1}{R^2} = \frac{8\pi\kappa}{3}(\varrho_{\text{matter}} + \varrho_{\text{dark matter}} + \varrho_{\text{dark energy}}) \quad (5.6.2)$$

Now, the critical density [9] is

$$\varrho_{\text{critical}} := \frac{3H^2}{8\pi\kappa} \quad (5.6.3)$$

So,

$$\frac{1}{\varrho_{\text{critical}}}(\varrho_{\text{matter}} + \varrho_{\text{dark matter}} + \varrho_{\text{dark energy}}) = 1 + \frac{1}{\dot{R}^2} \quad (5.6.4)$$

or, in terms of density parameter [9],

$$\Omega_{\dots} := \frac{\varrho_{\dots}}{\varrho_{\text{critical}}} \quad (5.6.5)$$

$$\Omega = \Omega_{\text{matter}} + \Omega_{\text{dark matter}} + \Omega_{\text{dark energy}} = 1 + \frac{1}{\dot{R}^2} \quad (5.6.6)$$

Again,

$$\dot{R} = \frac{4}{\sqrt{\kappa\nu}} \sin^3 \nu t \cos \nu t \quad (5.6.7)$$

so that

$$\Omega = 1 + \frac{\kappa\nu^2}{16 \sin^6 \nu t \cos^2 \nu t} = 1 + \frac{(t_{\text{Planck}}\nu)^2}{16 \sin^6 \nu t \cos^2 \nu t} = 1 + \frac{0.366 \times 10^{-124}}{\sin^6 \nu t \cos^2 \nu t} \quad (5.6.8)$$

6 On justification of the concept of compenson

6.1 Energy-momentum compenson

The energy-momentum compenson Θ_{μ}^0 compensates for quantum jumps of T_{μ}^0 in (1.3.3), (1.3.4).

6.2 Stress compenson

We have (2.1.1):

$$g_{ij}(t, \underline{x}) = -R^2(t)h_{ij}(t, \underline{x}) \quad \text{with } R(t) \text{ preassigned} \quad (6.2.1)$$

In order that this make sense, i.e., be nontrivial, there should be an additional equation (aside from the Einstein equations) for h_{ij} and, accordingly, a stress compenson function compensating for the equation.

7 Stress compenson specified

7.1 The spatial Einstein equation with compenson

The spatial Einstein equation with compenson is

$$G_i^j = 8\pi\kappa(T_i^j + \Theta_i^j) \quad (7.1.1)$$

7.2 Pressure compenson

We shall accept the simplest form of the stress compenson:

$$\Theta_i^j = \vartheta \delta_i^j \quad (7.2.1)$$

the function (i.e., scalar) ϑ will be called the pressure compenson:

$$\vartheta = -p_{\text{compenon}} \quad (7.2.2)$$

8 Pressure compenson eliminated

8.1 The extended spatial Einstein equation

The extended spatial Einstein equation takes the form

$$G_i^j = 8\pi\kappa(T_i^j + \vartheta \delta_i^j) \quad (8.1.1)$$

This is a system of 6 equations for metric components and the pressure compenson.

8.2 Eliminating pressure compenson

Let us express the Einstein tensor in (8.1.1) through the Ricci tensor:

$$R_i^j - \frac{1}{2}R_\mu^\mu \delta_i^j = 8\pi\kappa T_i^j + 8\pi\kappa \vartheta \delta_i^j \quad (8.2.1)$$

We have

$$R_l^l - \frac{3}{2}R_\mu^\mu = 8\pi\kappa T_l^l + 24\pi\kappa \vartheta \quad (8.2.2)$$

whence

$$\vartheta = \frac{1}{24\pi\kappa} \left(R_l^l - \frac{3}{2}R_\mu^\mu \right) - \frac{1}{3}T_l^l \quad (8.2.3)$$

Substitute (8.2.3) into (8.2.1):

$$R_i^j - \frac{1}{3}R_l^l \delta_i^j = 8\pi\kappa \left(T_i^j - \frac{1}{3}T_l^l \delta_i^j \right) \quad (8.2.4)$$

Let us introduce a traceless tensor

$$\bar{X}_i^j := X_i^j - \frac{1}{3}X_l^l \delta_i^j, \quad \bar{X}_l^l = 0 \quad (8.2.5)$$

So we have obtained the equation

$$\bar{R}_i^j = 8\pi\kappa \bar{T}_i^j \quad (8.2.6)$$

which represents a system of 5 equations for metric components.

9 Metric and the system of dynamical equations

9.1 Metric

Metric is of the form

$$ds^2 = dx^{02} + g_{ij}dx^i dx^j \quad (9.1.1)$$

It is convenient to represent metric also in the form

$$ds^2 = dt^2 - dl^2 \quad (9.1.2)$$

with

$$dl^2 = R^2(t)h_{ij}(\underline{x}, t)dx^i dx^j \quad (9.1.3)$$

9.2 The complete system of dynamical equations for metric

We have (2.3.9)

$$h^{ij}\ddot{h}_{ij} + \dot{h}^{ij}\dot{h}_{ij} = 0 \quad (9.2.1)$$

Again,

$$h^{il}h_{lj} = \delta_j^i, \quad \dot{h}^{il}h_{lj} + h^{il}\dot{h}_{lj} = 0 \quad (9.2.2)$$

whence

$$\dot{h}^{ij} = -h^{il}\dot{h}_{lk}h^{kj} \quad (9.2.3)$$

and, in view of (2.3.7),

$$\dot{h}^{ij}h_{ij} = 0 \quad (9.2.4)$$

Equation (9.2.1) takes the form

$$h^{ij}\ddot{h}_{ij} - h^{il}\dot{h}_{lk}h^{kj}\dot{h}_{ij} = 0 \quad (9.2.5)$$

Equations (8.2.6)

$$\bar{R}_i^j = 8\pi\kappa\bar{T}_i^j \quad (9.2.6)$$

and (9.2.5) form a complete system of 6 dynamical equations for 6 metric components h_{ij} .

10 The Christoffel symbols and Ricci tensor

10.1 The Christoffel symbols

Let us introduce

$$\beta_{ij} := R^2 h_{ij}, \quad \beta^{ij} = \frac{1}{R^2} h^{ij}, \quad \beta^{il}\beta_{lj} = h^{il}h_{lj} = \delta_j^i \quad (10.1.1)$$

and

$$\sigma_{ij} := \frac{\partial}{\partial t}\beta_{ij} = \dot{\beta}_{ij} \quad (10.1.2)$$

The Christoffel symbols are (here and further we exploit [8])

$$\begin{aligned}\Gamma_{00}^0 &= 0, \quad \Gamma_{00}^i = 0, \quad \Gamma_{0i}^0 = 0 \\ \Gamma_{ij}^0 &= \frac{1}{2}\sigma_{ij}, \quad \Gamma_{0j}^i = \frac{1}{2}\sigma_j^i \\ \Gamma_{jl}^i &= \lambda_{jl}^i\end{aligned}\tag{10.1.3}$$

where

$$\sigma_j^i = \beta^{il}\sigma_{lj} = h^{il}\dot{h}_{lj} + 2\frac{\dot{R}}{R}\delta_j^i\tag{10.1.4}$$

and

$$\lambda_{jl}^i = \frac{1}{2}\beta^{im}(\beta_{ml,j} + \beta_{jm,l} - \beta_{jl,m}) = \frac{1}{2}h^{im}(h_{ml,j} + h_{jm,l} - h_{jl,m})\tag{10.1.5}$$

10.2 The Ricci tensor

The Ricci tensor components are these:

$$R_0^0 = R_{00} = -\frac{1}{2}\frac{\partial}{\partial t}\sigma_l^l - \frac{1}{4}\sigma_l^k\sigma_k^l\tag{10.2.1}$$

$$R_i^0 = R_{0i} = \frac{1}{2}(\sigma_{i;l}^l - \sigma_{l;i}^l)\tag{10.2.2}$$

$$R_{ij} = \frac{1}{2}\frac{\partial}{\partial t}\sigma_{ij} + \frac{1}{4}(\sigma_{ij}\sigma_l^l - 2\sigma_i^l\sigma_{jl}) + Q_{ij}\tag{10.2.3}$$

$$R_i^j = -\frac{1}{2\sqrt{|\beta|}}\frac{\partial}{\partial t}(\sqrt{|\beta|}\sigma_i^j) - Q_i^j\tag{10.2.4}$$

Here

$$Q_{ji} = Q_{ij} = \frac{\partial\Gamma_{ij}^l}{\partial x^l} - \frac{\partial\Gamma_{il}^j}{\partial x^i} + \Gamma_{ij}^l\Gamma_{lm}^m - \Gamma_{il}^m\Gamma_{jm}^l\tag{10.2.5}$$

$$Q_i^j = \beta^{jk}Q_{ki} = \frac{1}{R^2}h^{jk}Q_{ki}\tag{10.2.6}$$

Next,

$$\sigma_l^l = h^{lk}\dot{h}_{lk} + 2\frac{\dot{R}}{R}\delta_l^l = 6\frac{\dot{R}}{R}\tag{10.2.7}$$

whence

$$\sigma_{l;i}^l = \sigma_{i;l}^l = 0\tag{10.2.8}$$

$$\frac{\partial}{\partial t}\sigma_l^l = 6\left(\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2}\right)\tag{10.2.9}$$

We obtain

$$\sigma_l^k\sigma_k^l = k^{km}\dot{h}_{ml}h^{ln}\dot{h}_{nk} + 12\frac{\dot{R}^2}{R^2}\tag{10.2.10}$$

and

$$\sigma_{i;l}^l = (h^{lk}\dot{h}_{ki})_{,l} - \Gamma_{il}^m h^{lk}\dot{h}_{km} + \Gamma_{ml}^l h^{mk}\dot{h}_{ki}\tag{10.2.11}$$

Again, by (10.1.1)

$$|\beta| = R^6 |h| \quad (10.2.12)$$

so that, in view of (2.3.5), (10.2.4) reduces to

$$R_i^j = -\frac{1}{2R^3} \frac{\partial}{\partial t} (R^3 \sigma_i^j) - Q_i^j \quad (10.2.13)$$

11 The metrodynamical equation.

The Haar measure condition as an integral of motion

11.1 The metrodynamical equation

Let us return to (9.2.6):

$$\bar{R}_i^j = 8\pi \varkappa \bar{T}_i^j \quad (11.1.1)$$

By (10.2.13)

$$\bar{R}_i^j = -\frac{1}{2R^3} \frac{\partial}{\partial t} (R^3 \bar{\sigma}_i^j) - \bar{Q}_i^j \quad (11.1.2)$$

Again,

$$\bar{\sigma}_i^j = \sigma_i^j - \frac{1}{3} \sigma_i^l \delta_i^j = h^{jk} \dot{h}_{ki} \quad (11.1.3)$$

so that

$$\bar{R}_i^j = -\frac{3}{2} \frac{\dot{R}}{R} (h^{jk} \dot{h}_{ki}) - \frac{1}{2} (\dot{h}^{jk} \dot{h}_{ki} + h^{jk} \ddot{h}_{ki}) - \bar{Q}_i^j \quad (11.1.4)$$

Next, substituting (9.2.3) for \dot{h}^{jk} , we represent (11.1.1) as

$$h^{jk} \left[\frac{1}{2} \ddot{h}_{ki} - \frac{1}{2} \dot{h}_{km} h^{ml} \dot{h}_{li} + \frac{3}{2} \frac{\dot{R}}{R} \dot{h}_{ki} \right] + \bar{Q}_i^j = -8\pi \varkappa \bar{T}_i^j \quad (11.1.5)$$

or

$$\ddot{h}_{ij} + 3 \frac{\dot{R}}{R} \dot{h}_{ij} - \dot{h}_{im} h^{mn} \dot{h}_{nj} + 2 h_{il} \bar{Q}_j^l = -16\pi \varkappa h_{il} \bar{T}_j^l \quad (11.1.6)$$

Now, let us return to (8.2.5)

$$\bar{X}_i^j := X_i^j - \frac{1}{3} X_l^l \delta_i^j, \quad \bar{X}_l^l = 0 \quad (11.1.7)$$

and introduce

$$\bar{X}_{ij} := X_{ij} - \frac{1}{3} h_{ij} h^{mn} X_{mn}, \quad X_{ij} = X_{ji}, \quad h^{ij} \bar{X}_{ij} = 0 \quad (11.1.8)$$

We find

$$h_{il} \bar{Q}_j^l = \frac{1}{R^2} \beta_{il} \bar{Q}_j^l = \frac{1}{R^2} \bar{Q}_{ij} \quad (11.1.9)$$

and (see [8])

$$h_{il} \bar{T}_j^l = \frac{1}{R^2} \beta_{il} \bar{T}_j^l = -\frac{1}{R^2} g_{il} \bar{T}_j^l = -\frac{1}{R^2} \bar{T}_{ij} \quad (11.1.10)$$

Thus, (11.1.6) takes a resultant form

$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \dot{h}_{im}h^{mn}\dot{h}_{nj} + \frac{2}{R^2}\bar{Q}_{ij} = 16\pi\kappa\frac{1}{R^2}\bar{T}_{ij} \quad (11.1.11)$$

This is the metrodynamical equation—a dynamical equation for metric components.

11.2 The Haar measure condition as an integral of motion of the metrodynamical equation

From the metrodynamical equation follows

$$h^{ji}\ddot{h}_{ij} - h^{ji}\dot{h}_{im}h^{mn}\dot{h}_{nj} + 3\frac{\dot{R}}{R}(h^{ji}\dot{h}_{ij}) = 0 \quad (11.2.1)$$

or, by using (9.2.3),

$$h^{ji}\ddot{h}_{ij} + \dot{h}^{ji}\dot{h}_{ij} + 3\frac{\dot{R}}{R}(h^{ji}\dot{h}_{ij}) = 0 \quad (11.2.2)$$

i.e.,

$$\frac{\partial}{\partial t}(h^{ji}\dot{h}_{ij}) + 3\frac{\dot{R}}{R}(h^{ji}\dot{h}_{ij}) = 0 \quad (11.2.3)$$

By the Haar measure condition (2.3.7)

$$h^{ji}\dot{h}_{ij} = 0 \quad (11.2.4)$$

so that (11.2.3) is an identity. However, we may reverse the treatment, i.e., treat (11.2.3) as an equation. Then we obtain

$$[\partial(h^{ji}\dot{h}_{ij})/\partial t]/(h^{ji}\dot{h}_{ij}) + 3[dR/dt]/R = 0 \quad (11.2.5)$$

i.e.,

$$\frac{\partial}{\partial t} \ln[(h^{ji}\dot{h}_{ij})R^3] = 0 \quad (11.2.6)$$

whence

$$h^{ji}\dot{h}_{ij} = \frac{f(\underline{x})}{R^3(t)} \quad (11.2.7)$$

Equation (11.2.4) implies

$$f(\underline{x}) = 0 \quad (11.2.8)$$

Thus, the Haar measure condition may be regarded as an integral of motion for the metrodynamical equation.

12 Compenson determined

12.1 Pressure compenson

The pressure compenson is (8.2.3)

$$\vartheta = \frac{1}{24\pi\kappa} \left(R_l^l - \frac{3}{2} R_\mu^\mu \right) - \frac{1}{3} T_l^l = -\frac{1}{24\pi\kappa} \left[\frac{1}{2} (R_l^l + 3R_0^0) + 8\pi\kappa T_l^l \right] \quad (12.1.1)$$

By (10.2.13)

$$R_l^l = -\frac{1}{2R^3} \frac{\partial}{\partial t} (R^3 \sigma_l^l) - Q_l^l \quad (12.1.2)$$

and by (10.2.6)

$$Q_l^l = \frac{1}{R^2} h^{lk} Q_{kl} \quad (12.1.3)$$

Using (10.2.1), (10.2.7), (10.2.9) we obtain

$$\vartheta = \frac{1}{8\pi\kappa} \left\{ \left[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{6R^2} h^{lk} Q_{kl} + \frac{1}{8} h^{lm} \dot{h}_{mk} h^{kn} \dot{h}_{nl} \right] - \frac{8\pi\kappa}{3} T_l^l \right\} \quad (12.1.4)$$

Notation:

$$(\dot{h})_{nl} := \dot{h}_{nl}, \quad (\dot{h})^{ln} := h^{lm} (\dot{h})_{mn} h^{kn}, \quad (\dot{h})^{ln} \neq \dot{h}^{ln} \quad (12.1.5)$$

(12.1.4) takes the form

$$\vartheta = \frac{1}{8\pi\kappa} \left\{ \left[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{6R^2} h^{lk} Q_{kl} + \frac{1}{8} (\dot{h})^{ln} (\dot{h})_{nl} \right] - \frac{8\pi\kappa}{3} T_l^l \right\} \quad (12.1.6)$$

12.2 Energy compenson

From the extended Einstein equation (1.3.3)

$$R_0^0 - \frac{1}{2} R_\mu^\mu = 8\pi\kappa (T_0^0 + \Theta_0^0) \quad (12.2.1)$$

follows the expression for the energy compenson

$$\Theta_0^0 = \frac{1}{16\pi\kappa} (R_0^0 - R_l^l) - T_0^0 \quad (12.2.2)$$

We obtain

$$\Theta_0^0 = \frac{1}{8\pi\kappa} \left\{ \left[3\frac{\dot{R}^2}{R^2} + \frac{1}{2R^2} h^{lk} Q_{kl} - \frac{1}{8} (\dot{h})^{lk} (\dot{h})_{kl} \right] - 8\pi\kappa T_0^0 \right\} \quad (12.2.3)$$

12.3 Momentum compenson

From the extended Einstein equation (1.3.4)

$$R_i^0 = 8\pi\kappa(T_i^0 + \Theta_i^0) \quad (12.3.1)$$

follows the expression for the momentum compenson

$$\Theta_i^0 = \frac{1}{8\pi\kappa} R_i^0 - T_i^0 \quad (12.3.2)$$

By (10.2.2), (10.2.8), (10.2.11)

$$R_i^0 = \frac{1}{2} \left[(h^{lk} \dot{h}_{ki})_{,l} - \Gamma_{il}^m h^{lk} \dot{h}_{km} + \Gamma_{ml}^l h^{mk} \dot{h}_{ki} \right] \quad (12.3.3)$$

Notation:

$$(\dot{h})_i^j := h^{jk} (\dot{h})_{ki}, \quad (\dot{h})_i^j \neq \dot{h}_i^j = 0 \quad (12.3.4)$$

Thus,

$$\Theta_i^0 = \frac{1}{8\pi\kappa} \left\{ \frac{1}{2} \left[(\dot{h})_{i,l}^l - \Gamma_{il}^m (\dot{h})_m^l + \Gamma_{ml}^l (\dot{h})_i^m \right] - 8\pi\kappa T_i^0 \right\} \quad (12.3.5)$$

12.4 Quantum jumps and compenson

From (12.1.6), (12.2.3), (12.3.5) it follows that under quantum jumps of the energy-momentum tensor

$$T_\mu^\nu = (\Psi, \hat{T}_\mu^\nu \Psi) \quad (12.4.1)$$

the quantities

$$\frac{1}{3} T_l^l + \theta, \quad T_0^0 + \Theta_0^0, \quad T_i^0 + \Theta_i^0 \quad (12.4.2)$$

remain continuous.

Next, let us return to (9.2.1)

$$h^{ij} \ddot{h}_{ij} + \dot{h}^{ij} \dot{h}_{ij} = 0 \quad (12.4.3)$$

Since

$$h^{ij} \ddot{h}_{ij} + \dot{h}^{ij} \dot{h}_{ij} = \frac{\partial}{\partial t} (h^{ij} \dot{h}_{ij}) \quad (12.4.4)$$

and according to (11.2.4)

$$h^{ij} \dot{h}_{ij} = 0 \quad (12.4.5)$$

the quantity

$$h^{ij} \ddot{h}_{ij} \quad (12.4.6)$$

is continuous under quantum jumps.

13 The isotropic universe. Background and deviation

13.1 The isotropic universe

Let us return to the isotropic universe, in which it holds that

$$A_i^0 = 0, \quad A_i^j = 0 \text{ for } i \neq j, \quad A_1^1 = A_2^2 = A_3^3 \quad (13.1.1)$$

From (11.1.7) follows

$$\bar{A}_i^j = 0 \quad (13.1.2)$$

and by (11.1.9), (11.1.10)

$$\bar{T}_{ij} = 0, \quad \bar{Q}_{ij} = 0 \quad (13.1.3)$$

The extended Einstein equations are

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = 8\pi\kappa(T_1^1 + \vartheta) \quad (13.1.4)$$

$$3\left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2}\right) = 8\pi\kappa(T_0^0 + \Theta_0^0) \quad (13.1.5)$$

The Haar measure condition is satisfied identically. The metrodynamical equation in the form of (11.1.6) reduces to an identity.

The expression for the pressure compenson (12.1.6) reduces to

$$\vartheta = \frac{1}{8\pi\kappa} \left\{ \left[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{6R^2} h^{mn} Q_{mn} \right] - \frac{8\pi\kappa}{3} T_l^l \right\} \quad (13.1.6)$$

From (13.1.6), (13.1.4) follows

$$h^{mn} Q_{mn} = 6 \quad (13.1.7)$$

and by (12.1.3)

$$Q_l^l = \frac{6}{R^2} \quad (13.1.8)$$

13.2 Background and deviation

Let us introduce the notion of a background and deviation with the background corresponding to the isotropic universe. So,

$$(T + \Theta) = (T + \Theta)_{\text{background}} + (T + \Theta)_{\text{deviation}} \quad (13.2.1)$$

$$8\pi\kappa(T_1^1 + \vartheta)_{\text{background}} = 2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \quad (13.2.2)$$

$$8\pi\kappa(T_0^0 + \Theta_0^0)_{\text{background}} = 3\left(\frac{\dot{R}^2}{R^2} + \frac{1}{R^2}\right) \quad (13.2.3)$$

$$\begin{aligned} A_{\text{background } i}^0 &= 0, \quad \bar{A}_{\text{background } ij} = 0 \\ A_i^0 &= A_{\text{deviation } i}^0, \quad \bar{A}_{ij} = \bar{A}_{\text{deviation } ij}, \quad A = T, Q \end{aligned} \quad (13.2.4)$$

13.3 Background eliminated

The metrodynamical equation (11.1.11) takes the form

$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - (\dot{h})^{in}(\dot{h})_{nj} + \frac{2}{R^2}\bar{Q}_{\text{deviation } ij} = 16\pi\kappa\frac{1}{R^2}\bar{T}_{\text{deviation } ij} \quad (13.3.1)$$

We rewrite equation (12.2.3) as

$$\left[\frac{1}{2}Q_m^m - \frac{3}{R^2}\right] - \frac{1}{8}(\dot{h})^{lk}(\dot{h})_{kl} = 8\pi\kappa(T + \Theta)_{\text{deviation } 0} \quad (13.3.2)$$

and equation (12.3.5) as

$$\frac{1}{2}\left[(\dot{h})_{i,l}^l - \Gamma_{il}^m(\dot{h})_m^l + \Gamma_{ml}^l(\dot{h})_i^m\right] = 8\pi\kappa(T + \Theta)_{\text{deviation } i} \quad (13.3.3)$$

Next, (12.1.6) may be written as

$$\vartheta = \frac{1}{8\pi\kappa} \left\{ \left[\left(\frac{1}{6}Q_l^l - \frac{1}{R^2} \right) + \frac{1}{8}(\dot{h})^{ln}(\dot{h})_{nl} \right] + \left[\left(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) - \frac{8\pi\kappa}{3}T_l^l \right] \right\} \quad (13.3.4)$$

or

$$\vartheta = \frac{1}{8\pi\kappa} \left\{ \left[\left(\frac{1}{6}Q_l^l - \frac{1}{R^2} \right) + \frac{1}{8}(\dot{h})^{ln}(\dot{h})_{nl} \right] + 8\pi\kappa\theta_{\text{background}} - \frac{8\pi\kappa}{3}T_{\text{deviation } l}^l \right\} \quad (13.3.5)$$

so that

$$\vartheta_{\text{deviation}} = \frac{1}{8\pi\kappa} \left\{ \left[\left(\frac{1}{6}Q_l^l - \frac{1}{R^2} \right) + \frac{1}{8}(\dot{h})^{ln}(\dot{h})_{nl} \right] - \frac{8\pi\kappa}{3}T_{\text{deviation } l}^l \right\} \quad (13.3.6)$$

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